Uncertainty and Beauty in the Brain Sciences, Illustrated by Torus Graphs

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Abstract:

Much has been written about “beauty” in mathematics. After a few quick remarks on that heady subject I will pivot to the question of where we might find such aesthetic pleasure in statistics, beginning with Bayesian inference. I will then float back down to Earth by presenting some recent work on multivariate phase coupling in neurophysiology.

Interdependence among multiple brain regions is of great interest, and a leading theory is that communication across areas may be facilitated by neural oscillations. One way to establish association of multiple oscillating signals is by showing their phases to be correlated (across repeated measurements), in the sense that they tend to advance or recede together. I will describe an example in which 24 recordings have been made simultaneously from 4 regions of the brain, repeatedly, during a memory task (data are from the lab of Earl Miller at MIT). To analyze such multivariate angular data, it would be possible to check all relevant pairs of signals using an angular analogue of correlation, known as Phase Locking Value (PLV). While useful, PLV is unable to reveal multivariate dependencies: PLV is analogous to correlation, but with multiple angles it is desirable to have an analogue of partial correlation. We have developed a methodology that produces a functional connectivity graph based on the natural analogue to Gaussian graphical models, which we call torus graphs because angles lie on the circle and the product of circles is a torus. (Our paper may be found on my website.) Torus graphs generalize several more restrictive approaches that have appeared in various scientific literatures. We show that they have nice properties and, in the data, we find phase relationships that PLV obscures. Interestingly, dependence in torus graphs can be quite different than in Gaussian graphs: for example, in the bivariate Gaussian case, a single scalar, correlation, can describe both positive and negative association; in a 2-dimensional torus graph a complete description of association requires 2 complex numbers. The torus graph framework should be useful in studies of functional connectivity among multiple oscillating brain areas. It is also an example of beauty in statistics.