Combining Estimates from Related Surveys via Bivariate Models

(Application: using ACS estimates to improve estimates from smaller U.S. surveys)

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Disclaimer:

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Introduction

- Investigate the potential of using bivariate models to borrow strength from estimates from a large survey to improve related estimates from smaller surveys.
- Motivation: "Large survey" is the Census Bureau's American Community Survey (ACS), the largest U.S. household survey.
- Approach is simple and requires no covariates from auxiliary information.
- Real examples show that large reductions in standard errors of estimates are possible.

ACS: The Largest U.S. Household Survey

American Community Survey (ACS)

- Conducted annually (data collected throughout the year) and has replaced the decennial census long form sample.
- Samples approximately 3.5 million addresses each year.
- Encompasses a broad range of topics: demographic, income, health insurance, employment, disabilities, occupations, housing, education, veteran status, etc.
- Produces estimates annually based on 1 or 5 years of data.

Three Smaller U.S. Surveys

- **Survey of Income and Program Participation (SIPP) Disability Module**
  - Approx. 37,000 households and 70,000 persons in 2008 panel.
  - Detailed questions about many different aspects of disability.
- **National Health Interview Survey (NHIS)**
  - About 110,000 persons in Family Core component, 2013.
  - Questions about a broad range of health topics asked in personal household interviews.
  - Estimates used to track health status, health care access, and progress toward achieving national health objectives
- **Current Population Survey (CPS) Annual Social and Economic Supplement**.
  - Samples about 100,000 addresses.
  - Provides official national estimates of income and poverty.

Four Applications

1. **SIPP estimates of U.S. state disability rates**.
   ACS variable: Estimate of state disability rates (types of disabilities and the time frames differ from SIPP).
2. **NHIS estimates of U.S. state uninsured rates**.
   ACS variable: Estimate of U.S. state uninsured rates (questions asked and the mode of survey delivery and design differ from NHIS).
3. **CPS estimates of per capita expenditure on health insurance premiums by state**
   ACS variable: Estimated per capita income by state.
4. **ACS 1-yr estimates (of anything! Take county rates of children in poverty to illustrate)**
   2nd variable: Corresponding previous ACS 5-yr estimates (larger sample size, but less current).

Univariate Gaussian Shrinkage Model for Survey Estimates

- For m small areas:

$$y_i = Y_i + e_i \qquad i = 1, \ldots, m$$
$$Y_i = \mu + u_i$$

- y_i is the direct survey estimate of Y_i, the population characteristic of interest for area i.
- e_i is the sampling error in y_i, generally assumed to be $N(0, v_i)$, independent with v_i known.
- u_i is the area i random effect, usually assumed to be *i.i.d.* $N(0, \sigma_u^2)$ and independent of the e_i.

Shrinkage Estimation (Stein 1956, Carter and Rolph 1974)

- Best linear predictor of Y_i (μ and σ^2 known):

$$\hat{Y}_i = (1 - \gamma_i) y_i + \gamma_i \mu$$

where

$$\gamma_i = \frac{v_i}{v_i + \sigma_u^2}$$

- Weighted average $\hat{Y}_i$ "shrinks" the direct estimate y_i towards the overall mean μ.
- The smaller is the sampling variance v_i the more weight is placed on the direct survey estimate y_i.
- Parameters unknown: estimate by ML or REML, or take Bayesian approach.
- Fay and Herriot (1979) extended the approach to shrink y_i towards a regression mean $\mu_i = x_i' \beta$, and applied this approach to small area estimation.

Bivariate Gaussian Model

$$y_{1i} = Y_{1i} + e_{1i} = (\mu_1 + u_{1i}) + e_{1i}, \quad i = 1, \ldots, m.$$
$$y_{2i} = Y_{2i} + e_{2i} = (\mu_2 + u_{2i}) + e_{2i}$$

$$\begin{bmatrix} u_{1i} \\ u_{2i} \end{bmatrix} \overset{i.i.d}{\sim} N(0, \boldsymbol{\Sigma}), \quad \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$$

$$\begin{bmatrix} e_{1i} \\ e_{2i} \end{bmatrix} \overset{i.i.d}{\sim} N(0, \mathbf{V}_i), \quad \mathbf{V}_i = \begin{bmatrix} v_{11} & 0 \\ 0 & \sigma_{22} \end{bmatrix}$$

- y_{1i} is the direct estimate of the quantity of interest Y_{1i}, and y_{2i} is the direct estimate from another survey of a related quantity Y_{2i}.
- Note that $\mathbf{V}_i$ assumes the sampling errors e_{1i} and e_{2i} are uncorrelated. This can be generalized.
- The alternative of simply including y_{2i} as a regression covariate in the model would ignore their sampling errors!

Estimation/Inference for Model Parameters

- Unknown parameters: $\mu_1, \mu_2, \sigma_{11}, \sigma_{22}$, and σ_{12} or $\rho = \sigma_{12}/\sqrt{\sigma_{11}\sigma_{22}}$.
- Sampling variances v_{1i} and v_{2i} are treated as known (really estimated using survey microdata).
- Can estimate unknown parameters by ML or REML.
- We shall use a Bayesian approach with flat priors on $\mu_1, \mu_2, \sigma_{11} > 0, \sigma_{22} > 0$ and $\rho \in (-1, 1)$.
- Approach was implemented in JAGS.

Prediction When Model Parameters are Known

In matrix notation $\mathbf{y}_i = \mathbf{Y}_i + \mathbf{e}_i = (\boldsymbol{\mu} + \mathbf{u}_i) + \mathbf{e}_i$

- $\hat{\mathbf{Y}}_i^{BP} = E(\mathbf{Y}_i|\mathbf{y}_i) = \boldsymbol{\mu} + \boldsymbol{\Sigma}(\boldsymbol{\Sigma} + \mathbf{V}_i)^{-1}(\mathbf{y}_i - \boldsymbol{\mu})$
- $MSE(\hat{\mathbf{Y}}_i^{BP}) = Var(\mathbf{Y}_i|\mathbf{y}_i) = \boldsymbol{\Sigma} - \boldsymbol{\Sigma}(\boldsymbol{\Sigma} + \mathbf{V}_i)^{-1}\boldsymbol{\Sigma}$
- We are interested in predicting Y_{1i} only, not Y_{2i}
- $\hat{Y}_{1i}^{BP}$ is a linear combination of μ_1, $(y_{1i} - \mu_1)$, and $(y_{2i} - \mu_2)$.

MSE % Reductions from Shrinkage Estimation

- direct estimation to univariate shrinkage:

$$100 \times \left\{ 1 - \frac{\text{Var}(Y_{1i}|y_{1i})}{v_{1i}} \right\}$$

(more reduction as v_{1i} increases)

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- univariate to bivariate shrinkage:

$$100 \times \left\{ 1 - \frac{\text{Var}(Y_{1i}|y_{1i}, y_{2i})}{\text{Var}(Y_{1i}|y_{1i})} \right\}$$

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Application I: 2010 Disability Rates for U.S. States: SIPP borrowing from ACS

y_{1i} = SIPP disability estimate, y_{2i} = ACS disability estimate

Smoothing of SIPP direct sampling variance estimates is applied.

$\hat{\rho} = .\mathbf{82}$

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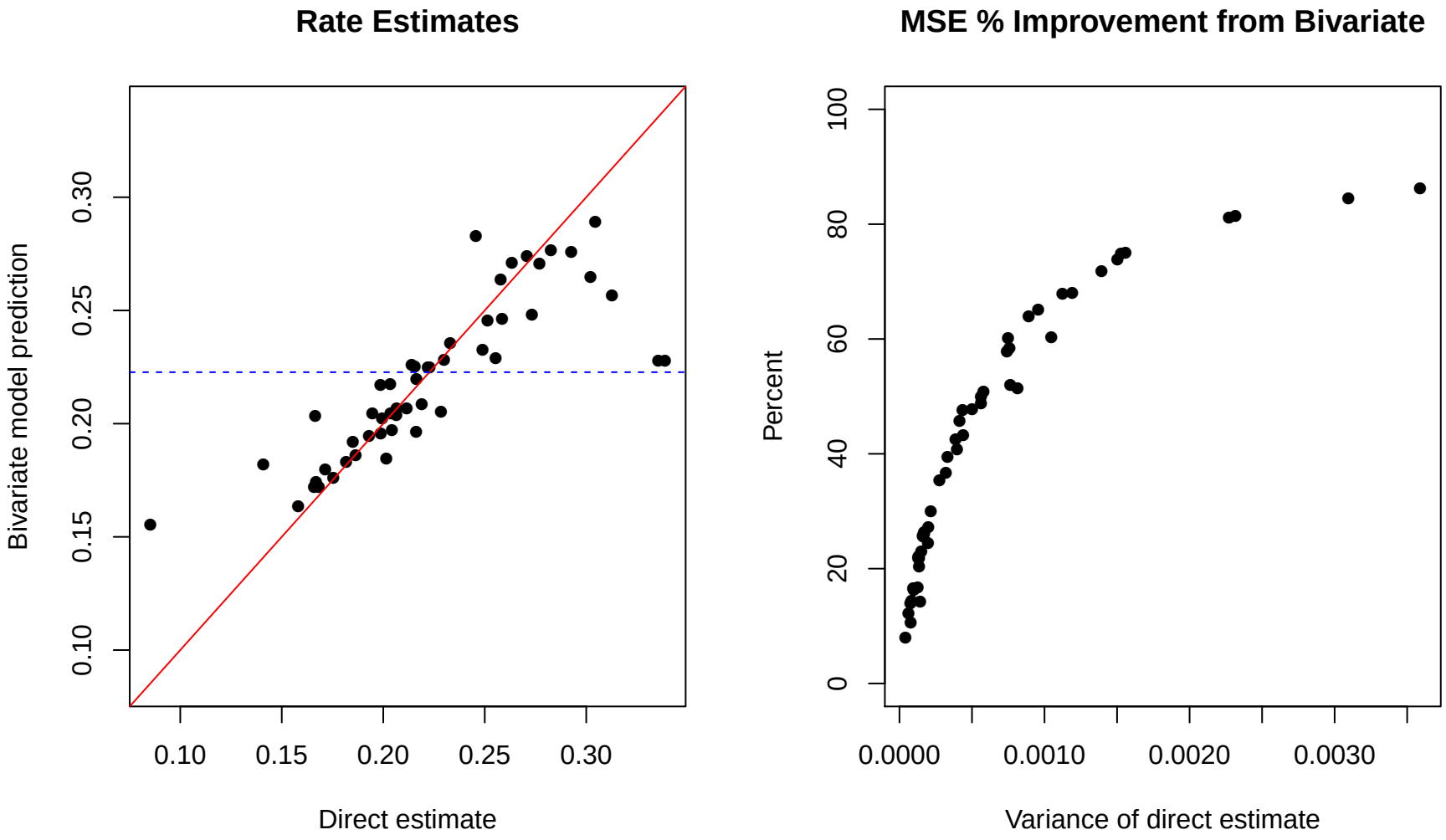
- The MSE decrease from bivariate vs. univariate model is $6\% - 59\%$ with a median of 29%
- The MSE decrease from bivariate vs. direct is $\mathbf{8 - 86\%}$, **with a median decrease of 43%**

Disability Rates for U.S. States, 2014

Bivariate model for SIPP and ACS estimates



Application II: 2013 Health Insurance Coverage Rates for U.S. States: NHIS Borrowing from ACS

y_{1i} = NHIS estimate of health insurance coverage (from National Center for Health Statistics)
y_{2i} = ACS estimate of health insurance coverage

Estimates published for only 43 states "due to considerations of sample size and precision."

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- MSE decrease UNI vs. Direct: $1\% - 16\%$, median $= 10\%$

Using bivariate model might allow publication of estimates for states that would otherwise be excluded (?)

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- MSE decrease UNI vs. Direct: $1\% - 16\%$, median $= 10\%$
- MSE decrease BIV vs. UNI: $16\% - 67\%$, median $= 54\%$
- MSE **decrease BIV vs. Direct: 19 – 72%, median = 60%**!

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Application III: 2012 Per Capita Expenditures for Health Insurance for U.S. States: CPS Borrowing from ACS

y_{1i} = CPS estimated per capita expenditure on health insurance premimums
y_{2i} = ACS per capita income estimate

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- MSE decrease UNI vs. Direct: $1\% - 55\%$, median $= 8\%$

More modest decreases overall, presumably because ρ and v_{1i}/σ_{11} are lower than in the previous examples.

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- MSE decrease UNI vs. Direct: $1\% - 55\%$, median $= 8\%$
- MSE decrease BIV vs. UNI: $-1.5\% - 28\%$, median $= 6\%$

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- MSE decrease UNI vs. Direct: $1\% - 55\%$, median $= 8\%$
- MSE decrease BIV vs. UNI: $-1.5\% - 28\%$, median $= 6\%$
- MSE decrease BIV vs. Direct: $2\% - 68\%$, median $= 14\%$

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Application IV: ACS 1-yr County Poverty Estimates Borrow from Previous ACS 5-yr County Poverty Estimates

y_{1i} = 2012 ACS estimated county rates of children in poverty

y_{2i} = 2007-2011 ACS estimated county child poverty rates

Note: Good covariates are available for modeling, but are not used here.

$\hat{\rho} = .\mathbf{94}$

- MSE decrease UNI vs. Direct: $0.4\% - 87\%$, median $= 32\%$

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- MSE decrease UNI vs. Direct: $0.4\% - 87\%$, median $= 32\%$
- MSE decrease BIV vs. UNI: $4\% - 65\%$, median $= 49\%$

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- MSE decrease UNI vs. Direct: $0.4\% - 87\%$, median $= 32\%$
- MSE decrease BIV vs. UNI: $4\% - 65\%$, median $= 49\%$
- **MSE decrease BIV vs. Direct: $4 - 91\%$, median $= 67\%$!!**

Concluding Remarks

- Bivariate model can achieve large MSE decreases by borrowing strength from ACS estimates to improve estimates from smaller surveys, provided ρ is high!
- Model is simple; key is the quality of the additional data source (ACS estimates) used for this purpose.
- In most of the examples (I, II, IV), the biggest part of the MSE decreases came from the univariate to bivariate shrinkage, not from the univariate shrinkage.
- Theoretical and empirical results show not much improvement when a larger survey borrows strength from a smaller one.