

**Instrumental Variables Type Estimation for Nonignorable Selection Bias
with a Continuous Instrument**

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INTRODUCTION

- **Economists (mostly) assume the existence of nonignorable selection bias**
- **Economists (mostly) use instrumental variables (IV) or some variant of it to adjust for that bias**
- **Economists (mostly) test for existence of nonignorable selection bias by specification tests based on a comparison of treatment effect estimates with and without IV**

- **Economists (mostly) like to use the framework of regression analysis to analyze the problem**
- **(Some) economists are moving toward less parametric estimation of the model**
- **Economists (mostly) admit that there is probably heterogeneity in treatment impact but do nothing about it except appeal to the LATE motivation of Imbens-Angrist (1994)**

- **(A few) economists use continuous instruments and attempt to map out the distribution of responses over as much of the participation probability support as possible**
- **Parametric version: Björklund-Moffitt (1987)**

A FORMULATION OF THE HETEROGENEOUS TREATMENT EFFECT MODEL

- **Potential outcomes in regression framework:**

α_i = treatment effect for individual i

$$= Y_{1i} - Y_{0i}$$

where Y_{D_i} = value of outcome if the individual participates

($D_i=1$) or does not ($D_i=0$)

- **Model:**

$$y_i = \beta_i + \alpha_i D_i$$

an equivalent formulation

$$D_i^* = k(Z_i, \delta_i)$$

$$D_i = 1(D_i^* \geq 0)$$

where Z_i is an exclusion restriction (potential instrument).

and where $\beta_i = Y_{0i}$ and $\alpha_i = Y_{1i} - Y_{0i}$

- **Ignoring for the moment what the object of estimation is, it looks as though there are two potential sources of bias from unobserved confounders:**

β_i may not be independent of D_i

α_i may not be independent of D_i

- **Indirectly, these are statements about independence of D_i from Y_{1i} and Y_{0i}**

Hence

$$\mathbf{E}(y_i | Z_i=z) = \mathbf{E}(\beta_i | Z_i=z) + \mathbf{E}(\alpha_i | D_i=1, Z_i=z) \mathbf{Prob}(D_i=1 | Z_i=z)$$

$$\mathbf{E}(D_i | Z_i=z) = \mathbf{Prob} [k(z, \delta_j) \geq 0]$$

Make the following assumptions:

A1. $E(\beta_i | Z_i=z) = \beta$

A2. $E(\alpha_i | D_i=1, Z_i=z) = g[E(D_i=1 | Z_i=z)]$

**A3. “Monotonicity” in effect of Z on D in D* equation
(for interpretation)**

With these assumptions, we have

$$y_i = \beta + g[F(Z_i)] F(Z_i) + \epsilon_i$$

$$D_i = F(Z_i) + v_i$$

where F is a proper probability function and where $E(\epsilon_i | gF) = E(v_i | F) =$

0 by construction

- **g is the treatment on the treated**
(mean of α_i for those observed to participate)
- **Average treatment effect is $g(1)$**
- **derivative of Y eqn. w.r.t. F is the marginal treatment effect (mean of α_i for those on the borderline of participation) (LATE is a discrete approximation to it)**

- **All functions (F and g) are nonparametrically identified at each Z point (g needs a normalization if the support of F in the data does not contain zero)**
- **Can use your favorite nonparametric method for g**
- **My preference: series estimation**
- **Can estimate the two equations with two-step methods or jointly with nonlinear least squares (my preference)**

- **Adding X: similar treatment gives:**

$$y_i = g[F(Z_i, X_i), X_i] F(Z_i, X_i) + h(X_i) + \epsilon_i$$

$$D_i = F(Z_i, X_i) + v_i$$

- **Three functions to estimate: $g(F, X)$, $h(X)$, and $F(Z, X)$**

Applications

- **Three from Public Health**

1. **Effect of hospital procedure (cathertization) on health outcomes**

(mortality). IV: distance to hospital.

(Newhouse, Annual Review of Public Health, 1998)

2. **Effect of smoking on health. IV: anti-smoking legislation.**

(Pearl, 1995)

3. **Effect of teen drinking on y. IV: state beer tax**

(W. Evans, 2003)

But all three have a continuous Z but estimate a single treatment effect

Table 1
Results of the Initial Estimation

	(1)	(2)	(3)	(4)	(5)
γ					
Constant	-30.9* (4.0)	-77.6* (8.0)	-7.1 (21.5)	-41.4 (37.0)	6.38 (78.7)
F	--	43.0 (6.0)	4.5 (19.2)	67.0 (68.3)	-94.9 (255.4)
Max(0,F-.25F)	--	--	--	--	113.4 (197.6)
Max(0,F-.50F)	--	--	--	-30.6 (51.9)	-11.9 (53.0)
Max(0,F-.75F)	--	--	--	--	18.0 (28.2)
λ					
Education	--	--	-3.16* (.91)	-1.56* (.94)	-1.86 (1.03)
Age	--	--	-1.91 (1.60)	2.41 (1.64)	2.42 (1.64)
Black	--	--	6.98* (3.28)	2.38 (3.48)	3.50 (3.72)
Family6	--	--	.74 (2.42)	-2.75 (2.63)	-1.86 (3.02)
Family	--	--	.98 (.92)	1.06 (1.12)	1.03 (.94)
Nonlabor	--	--	4.16* (1.62)	6.80* (1.93)	5.82* (2.26)
G	--	--	-.53 (.62)	-1.14* (.66)	-.98 (.69)

Table 1 (continued)

	(1)	(2)	(3)	(4)	(5)
β					
Constant	21.6* (3.2)	38.8* (5.2)	5.45 (6.89)	7.25 (7.53)	2.78 (10.4)
Education	.91* (.19)	.16* (.31)	2.60 (.43)	2.45* (.42)	2.60* (.48)
Age	1.68* (.31)	1.25* (.41)	1.20* (.53)	1.29* (.53)	1.30* (.54)
Black	-1.86* (.62)	-.14 (.84)	-3.93* (1.28)	-3.72* (1.31)	-4.23* (1.54)
Family6	.06 (.61)	.41 (.70)	-1.29 (1.14)	-1.61 (1.15)	-2.07 (1.41)
Family	-.71* (.19)	-.71* (.23)	-1.06* (.34)	-.98* (.34)	-.98* (.34)
Nonlabor	-.92* (.22)	-1.92* (.27)	-.96* (.31)	-1.02* (.44)	-.67 (.66)
G	-.27* (.14)	-.06 (.17)	-.18 (.25)	-.19 (.25)	-.27 (.27)
δ					
Constant	.62* (.19)	1.06* (.18)	.80* (.17)	.72* (.17)	.71* (.17)
Education	-.11* (.01)	-.16* (.01)	-.12* (.01)	-.12* (.01)	-.12* (.01)
Age	-.06* (.03)	-.06* (.03)	-.06* (.02)	-.05* (.02)	-.05* (.02)
Black	.29* (.05)	.31* (.04)	.34* (.04)	.36* (.04)	.36* (.04)

Table 1 (continued)

	(1)	(2)	(3)	(4)	(5)
Family6	.41* (.03)	.34* (.03)	.38* (.02)	.38* (.02)	.39* (.02)
Family	-.04* (.02)	-.01 (.02)	-.03* (.01)	-.02* (.01)	-.02 (.01)
Nonlabor	-.25* (.02)	-.21* (.02)	-.20* (.02)	-.21* (.02)	-.21* (.02)
G	.05* (.01)	.05* (.01)	.06* (.01)	.07* (.01)	.07* (.02)
η					
Admin	.90 (1.19)	.22 (.92)	-.41 (1.08)	-2.16* (1.04)	-1.85* (1.04)
Pctdenied	-.14 (.14)	-.08 (.10)	-.24* (.12)	-.31* (.11)	-.32* (.11)
Eligerror	-.03* (.01)	-.02* (.01)	-.02 (.01)	-.01 (.01)	-.01 (.01)

Notes:

Standard errors in parentheses

*: significant at the 10 percent level

Table 5

Parameter Estimates of g Function for Spline Specifications

	Z splines (P eqn only)	X-Z splines	
		P eqn only	y and P eqns ^a
Constant	-27.5* (12.4)	-20.7* (5.2)	-20.4* (9.37)
F	25.6* (8.7)	-.41 (4.03)	9.46* (5.70)
λ			
Education	-2.24* (.58)	--	--
Education12	--	-7.60* (1.86)	--
Age	.79 (1.39)	-2.29* (1.07)	--
Black	4.59* (2.29)	6.58* (1.64)	--
Family6	-1.72 (1.42)	.06 (.87)	--
Family	1.69* (.81)	--	--
Family>3	--	5.29* (1.57)	--
Nonlabor	5.05* (1.16)	2.08* (.66)	--
G	-1.09* (.51)	-.14 (.39)	--

Notes:

Standard errors in parentheses

*: significant at the 10% level

^a Coefficients on X splines shown in the Appendix [available upon request]